MANIFESTO OF STUDIES

A.A. 2023/2024

DOCTORAL SCHOOL IN MATHEMATICS

Approvato dal Collegio dei Docenti in Matematica in data 16.10.2023

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Manifesto of Studies

The Ph.D. degree in Mathematics is offered by the Department of Mathematics of the University of Trento in collaboration with the Department of Informatics of the University of Verona.
Our goal is to train students who wish to pursue a career in academic research and teaching, as well as in the private and public sectors.
Doctoral students take advanced courses in their areas of specialization, followed by a period of research and the preparation and defense of the doctoral thesis.
The Department of Mathematics of the University of Trento and the Department of Informatics of the University of Verona offer an advanced training in mathematics and its applications in the broadest sense.

Both Departments occupy a leading position in pure and applied mathematics, especially in Algebraic Geometry, Algebraic and Geometrical Methods in Cryptography, Calculus of Variations, Geometric Analysis and Riemannian Geometry, Probability Theory, Mathematical Statistics and Stochastic Processes, Dynamical Systems and Control Theory, Mathematical Logic and Theoretical Computer Science, Modellization and Scientific Computation, Mathematical Physics, Operations Research, Computational Biology, Quantum Science and Technology.

The School's curricula

The PhD Programme in Mathematics lasts three years and is articulated into five curricula:

a. General Mathematics;
This area focuses on one or more of the following research themes:

- Calculus of variations: analysis in metric spaces, geometric measure theory, variational convergences (Gamma-convergence), optimal transport.
- Geometric Analysis, Riemannian Geometry, Geometrical Flows.
- Non linear Partial Differential Equations (PDE): free boundary problems, models of hysteresis, asymptotic behavior and PDE homogenization, variational and topological methods, non linear equations of Ginzburg-Landau and Schrödinger.
- Analytic Geometry and Algebraic Geometry: Algebraic curves and moduli spaces. Surfaces of general type and moduli spaces. Manifolds of high dimensions: Mori theory, Fano varieties. Real algebraic geometry, complex and hypercomplex analysis. History of algebraic geometry. Mathematical visualization.
- Mathematical physics: foundational, analytical and geometrical aspects of quantum and relativistic theories.
- Dynamical Systems and Control Theory: existence, multiplicity, stability of periodic solutions of differential equations, Lagrangian and Hamiltonian systems, differential games and optimal control problems, viscosity solutions of Hamilton-Jacobi equations, mean field games, hybrid system optimizations.
- Stochastic processes: stochastic partial differential equations, functional integration and applications.
- Mathematical Statistics and Data Science. It will cover classical techniques of statistical inference, both under a frequentist and Bayesian paradigm, as well as modern techniques for complex and high-dimensional data. In particular, it will include topics from multivariate statistics, such as graphical models and their link to network science, robust statistics and statistical data depth.
- Mathematical Logic and Theoretical Computer Science: applications of non standard techniques (à la A. Robinson) in functional analysis, non classical logics, programming languages theory, type systems, static analysis, general and philosophical aspects, foundations, constructive mathematics and Hilbert's program.

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- Group theory, in particular permutation groups and finite p-groups, Lie groups and algebras, computational methods and applications in theoretical physics. Commutative and computational algebra, monomial algebras and associated combinatorial structures. Algorithms for algebraic and combinatorial invariants. Coding theory and cryptography. Tensor decomposition, secant varieties, algorithms and applications to complexity theory, quantum information, and data analysis. Representations of algebras, homological algebra.

b. Mathematical Modelling and Scientific Computing (MOMACS);
This area crosses the following research themes:

- Stochastic Processes: integral-differential equations and stochastic partial differential equations for the modeling of physical, biological, and financial phenomena.
- Numerical Methods for Partial Differential Equations: modeling of electromagnetic phenomena and (classic and quantum) fluid dynamics, approximation methods based on finite elements, boundary elements, differences or finite volumes.
- Approximation/numerical interpolation of multivariate functions: efficient methods and applications.
- Discrete Mathematics: modeling in operations research, graph theory, combinatorial optimization, and applications to computational biology.
- Optimal Control, Optimization: applications to decision science, image processing, cultural heritage.
- Mathematical and computational models in medicine: simulation of physiological and pathological mechanisms in the human organism, with special focus on the circulatory and lymphatic system and their interactions with the central nervous system.

c. Cryptography and coding theory;
This area focuses on several mathematical methods used in cryptography and in the theory of error correcting codes. More specifically, research is performed in the following topics:
- Algebraic methods: linear algebra, commutative algebra, algebraic combinatorics, computational algebra, Gröbner bases, number fields, group theory.
- Geometric methods: algebraic geometry, elliptic curves.
- Cryptographic protocols: design and formal proofs.

The proposed research problems range from purely theoretical classifications to problems close to industrial research. Industrial research can also be integrated with internships at leading companies within the field.

d. Mathematical and computational biology;
This area focuses on the vast field of applications of mathematical and computational models in biology.
Mathematical and computational models in medicine: simulation of physiological and pathological mechanisms in the human organism, with special focus on the circulatory and lymphatic system and their interactions with the central nervous system.

e. Mathematical applications to Quantum Science and Technologies.
This area focuses on theoretical and applied research on topics related to quantum physics in a broad sense (quantum mechanics, quantum information, quantum field theory) both from foundational-theoretical and applied points of view, involving advanced mathematical techniques in algebra, analysis, geometry, mathematical physics, computer science, and probability. Doctoral students can collaborate with the cross-disciplinary doctoral project “Quantum Science and Technology” within the Q@TN consortium.

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Coursework

According to the Doctorate Regulations, a candidate for the Ph.D. Degree in Mathematics must complete three courses, relevant to their research topic and taken from those presented in the Appendix of this document.
Doctoral students must present their study plan, in accordance with their advisor, no later than January 31, 2024. Possible other courses of interest, not listed in this document, can be introduced in the study plan if accompanied by a motivation for their choice and in agreement with the student's supervisor.
Any modification to the study plan, agreed with the supervisor, must be submitted no later than April 30, 2024.
Generally, all examinations must be completed within the end of July. In case of failure to do so, the student must present a motivation letter requesting for an extension. This must be submitted to the Director of the Doctoral School in Mathematics by **31 July 2024** (Doctoral School Committee dated 13/10/2021).
The School Committee, in accordance with the advisor, can ask for the presence in a student's study plan of any specific course which is regarded of particular interest for the scientific education of the student.
Doctoral students are required to participate in the "Math Bites" and "Doc in Progress" seminars, organized by the research groups of the Doctoral School, and to other activities (such as workshops, summer schools, and others) as proposed by their advisors.
Students of the curriculum "**e. Mathematical applications to Quantum Science and Technologies"** may choose their courses also among those in the Manifesto of the Q@TN Interdisciplinary Ph.D. School.

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Courses of the PhD School in Mathematics, a.y. 2023-2024

- **Title of course:** Model Theory
- **Lecturer and examiner:** Stefano Baratella
- **Course:** Borrowed from the Master's degree of Trento
- **Scientific sector:** MAT/01
- **Period:** 2nd semester 2023-2024
- **Venue:** University of Trento
- **Indicative number of academic hours:** 40
- **Assessment method:** Seminar
- **Contents:** Within mathematical logic, model theory is probably the field with stronger impact on the mathematical practice. In particular on algebra, complex and real algebraic geometry, analysis. This is an introductory course whose aim is twofold: to introduce some basic techniques in the field and to apply them to (re)prove a number of important results in various areas. The following topics will be covered: Categoricity. Ultraproducts. Los' theorem. Compactness and the Löwenheim-Skolem theorems. Ultraproducts in algebra and in analysis. The Ax-Grothendieck theorem. Quantifier elimination. Quantifier elimination for algebraically closed fields and for real closed fields. Model completeness. Hilbert's Nullstellensatz. Hilbert's 17th problem. Types. Saturated models. Elements of stability theory. Albeit not strictly required, prior knowledge of basic notions in mathematical logic is useful.

- **Title of course:** Advances in Mathematical Logic. Homotopy Type Theory
- **Lecturer and Examiner**: Iosif Petrakis
- **Course:** PhD course
- **Scientific Sector:** MAT/01
- **Period:** Nov-Dec 2023
- **Indicative number of academic hours:** 35
- **Assessment method:** Written exam
- **Contents:** Martin-Löf's type theory (MLTT) is a predicative modification of lambda-calculus with many applications to the theory of programming languages. The recent extension of MLTT with the axiom of univalence (UA) by the late Fields medalist Voevodsky reveals surprising connections between homotopy theory and logic. Homotopy Type Theory (HoTT) can be described as MLTT + UA + Higher Inductive Types. HoTT and Category Theory (CaT), are currently among the most actively studied mathematical frameworks for the logical foundations of mathematics and theoretical computer science. The following topics are planned to be covered: introduction to MLTT, function-extensionality axiom, the groupoid model of Hofmann and Streicher, Voevodsky's axiom of univalence, homotopy n-types, higher inductive types, the fundamental group of the higher circle, relations between HoTT and constructive mathematics, relations between HoTT and CaT.
- No prior knowledge of homotopy theory and type theory is required.
- Lecture notes will be available.
- **References**:
- E. Rijke: Introduction to Homotopy Type Theory, https://arxiv.org/abs/2212.11082, 2022, pre-publication version, which will be published by Cambridge University Press.
- I. Petrakis: Logic in Computer Science, Lecture notes, 2022. https://www.mathematik.uni-muenchen.de/~petrakis/LCS.pdf
- The Univalent Foundations Program Homotopy Type Theory: Univalent Foundations of Mathematics, Institute for Advanced Study, Princeton, 2013. https://homotopytypetheory.org/book/

Allegato n. 1

- **Title of course:** Methods in Representation Theory
- **Lecturer and Examiner:** Lidia Angeleri
- **Course:** Borrowed from the Master's degree of Verona
- **Scientific Sector:** MAT/02
- **Period:** November 2023 – January 2024
- **Venue:** University of Verona
- **Indicative number of academic hours:** 30
- **Assessment method:** The assessment is based on the seminar delivered in class and on the active participation in the entire course
- **Contents**: This is a reading course devoted to an introduction to geometric representation theory. We will see how certain algebraic varieties can be used to study algebras and their representations. The course is mainly based on the lecture notes "Geometry of representations of algebras" by W. Crawley-Boevey. Students actively participate in the course and deliver a seminar talk. The course also includes the lecture series "Geometric Invariant Theory for finite dimensional algebras", 11-15 December 2023, by Professor Charles Paquette of the Royal Military College of Canada.

- **Title of course:** Tensor Decomposition for big data analysis
- **Lecturer and Examiner**: Alessandra Bernardi
- **Course**: Borrowed from the Master's degree of Trento
- **Scientific Sector:** MAT/02
- **Period:** first semester 2023/2024
- **Venue**: University of Trento
- **Indicative number of academic hours:** 42 (For the topics covered before November, recorded video lessons will be available)
- **Assessment method:** frontal classes
- **Contents:** Examples of Big Data Problems. Geometric structures for modeling large-scale data: Segre, Veronese, Grassmannian, Varieties of secants. Ranks of a tensor and variants. Theoretical and numerical algorithms for tensor decomposition. Every year a specific masterclass is organized.

- **Title of course:** Advanced Cryptography
- **Lecturer:** Edoardo Ballico
- **Period:** Two months during the first semester 2023/2024
- **Venue:** University of Trento
- **Course:** Borrowed from the Master's degree of Trento
- **Indicative number of academic hours:** 20 hours for doctoral students (plus 70 hours of self-study)
- **Scientific Sector**: MAT/02, MAT/03
- **Examiner:** Edoardo Ballico
- **Assessment method:** Oral Exam
- **Contents:** The doctoral student should choose one of the following topics covered in both semesters:
  1) Goppa codes and codes obtained by Algebraic Geometric Methods (Codes from Algebraic Curves and evaluation codes from higher dimensional algebraic varieties).
  2) Elliptic curve cryptography.
  3) Post quantum cryptography using supersingular elliptic curves.
  4) Post quantum cryptography using lattices and their metrical properties.
  5) Computational Complexity.
  6) Boolean functions.

Allegato n. 1

- **Title of course:** An Introduction to Symplectic Geometry
- **Lecturer and Examiner:** Alessia Mandini
- **Course:** Borrowed from the Master's degree of Verona
- **Scientific Sector:** MAT/03
- **Period:** second semester of the academic year 2023/2024
- **Venue**: University of Verona
- **Indicative number of academic hours:** 48
- **Assessment method:** written or oral exam
- **Contents:** Symplectic geometry is a branch of differential geometry with roots in the geometric formulation of classical mechanics of the nineteenth century, known as Hamiltonian formalism. Its recents developments arise from its deep relation with different areas of Mathematics, for example Topology, Dynamics, Complex Geometry and Mathematical Physics.

In this course we will first give an introduction to symplectic geometry, with particular emphasis on the theory of group actions on symplectic manifolds, toric actions, symplectic reductions. In the latter part of the course we will give a brief introduction to Geometric Invariant Theory. GIT is a tool used for constructing quotient spaces in algebraic geometry, the most important such quotients being moduli spaces. We will study the basics of GIT, staying close to examples, with the aim to describe relation between symplectic and GIT quotients (Kempf-Ness theorem).

- **Title of course:** Geometry and Topology for Data Analysis
- **Lecturer:** Alessandro Oneto
- **Period:** Second Semester 2023/2024
- **Venue:** University of Trento
- **Course:** Borrowed from the Master's degree of Trento
- **Number of academic hours:** 42 hours (for master students), 30 hours (advanced part, for doctoral students)
- **Scientific Sector:** MAT/03
- **Examiner:** Alessandro Oneto
- **Assessment Method:** Seminar on research paper or project presentation
- **Contents:** The goal is to offer an introduction to computational algebraic topology and algebraic geometry, with the emphasis on applications to data analysis. The course will be divided in two main parts:
  - I) Introduction to algebraic topology and applications: simplicial complexes; definition of simplicial homology groups with coefficients in Z_2; induced maps in homology; relative homology; Exact sequences; Mayer-Vietoris exact sequence; simplicial cohomology; persistent homology and its applications.
  - II) Introduction to (numerical) algebraic geometry:
    - II.1) Introduction to algebraic geometry, with attention to applications in algebraic statistics: ideals and varieties; Zariski topology; projective space and projective varieties; algebraic statistical models.
    - II.2) Introduction to numerical algebraic geometry: Homotopy continuation methods for the solution of polynomial systems and applications to the study of algebraic statistical models.

- **Title of course:** Advanced Geometry
- **Lecturer:** Roberto Pignatelli
- **Period:** First Semester
- **Venue:** University of Trento
- **Course:** Borrowed from the Master's degree of Trento
- **Number of academic hours:** 63 for students of the Master's degree, 30 (advanced part) for doctoral students

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- **Scientific Sector**: MAT/03
- **Examiner:** Roberto Pignatelli
- **Assessment Method:** Seminar on research paper
- **Contents**: Elements of multilinear algebra. Complex manifolds and real manifolds with boundaries. Vector fields and Frobenius' Theorem. Orientability. Bundles. Differential forms. Integration. Stokes' theorem. De Rham cohomology and cohomology with compact support: definitions, computation tools (including the Mayer-Vietoris exact sequence, the Poincaré Lemma. the Poincaré duality and the Kuenneth formula) and applications. The Poincaré dual of a submanifold and applications.

- **Title of course:** Toric Geometry
- **Lecturers:** Elisa Postinghel, Luis Eduardo Solá Conde
- **Period:** February-May 2023/2024
- **Venue:** University of Trento
- **Course:** Borrowed from the Master's degree of Trento
- **Indicative number of academic hours:** 42 for students of the Master's degree, 30 (advanced part) for doctoral students
- **Scientific Sector:** MAT/03
- **Examiners:** Elisa Postinghel and Luis Eduardo Solá Conde
- **Assessment Method**: Seminar on project presentation
- **Contents:** The course is meant as an introduction to toric geometry for Master and PhD students. Toric geometry studies a special type of algebraic varieties that can be described in terms of discrete – combinatorial– data. Though small and manageable, the category of toric varieties is rich enough to be used to illustrate many of the basic concepts of algebraic geometry: affine and projective varieties, group actions, smoothness and singularity, divisors, line and vector bundles, etc, as well as some more advanced concepts of birational geometry such as ampleness, nefness, cones of divisors and curves, Mori dream spaces.

- **Title of course**: Advances in Mathematical Analysis: Rigidity phenomena for PDE problems arising in geometry and mathematical physics
- **Lecturers and Examiners**: Virginia Agostiniani, Stefano Borghini will be both lecturers (half lessons each) and examiners.
- **Course**: PhD course
- **Scientific Sector**: MAT/05
- **Period**: Second semester (indicatively, March-May 2024)
- **Venue**: University of Trento
- **Indicative number of academic hours**: 24
- **Assessment method**: Seminar on topics related to the course.
- **Contents**: In the first part of the course, we review some classical PDE problems in the Euclidean space:
  • Liouville-type Theorems.
  • Classical overdetermined boundary value problems: electrostatic potential and torsion.
  • Bernstein's Theorem on the characterization of minimal graphs.
  • Allen-Cahn Equation: De Giorgi's conjecture and partial answers.
  In the second part, we discuss how similar techniques can be applied to problems of interest in Riemannian Geometry and in General Relativity. A sample list of possible problems is:
  • Liouville-type theorem in nonnegatively curved manifolds.
  • Classification of static vacuum spacetimes without black holes.
  • Static Black Hole Uniqueness Theorem for zero cosmological constant. Partial results for positive and negative cosmological constant.

Allegato n. 1

- **Title of course:** Advanced in Theory of distributions and applications
- **Lecturer and Examiner**: Brunetti Romeo
- **Course:** PhD course
- **Scientific Sector:** MAT/05
- **Period:** March-June 2024
- **Venue**: University of Trento
- **Indicative number of academic hours:** 40
- **Assessment method:** Oral examination
- **Contents:**
  1. Theory of distributions
     1.a Smooth functions
     1.b Regular distributions
     1.c Order of a Distribution
     1.d Support of a distribution
     1.e Compactly supported distributions
     1.f Operations on Distributions
     1.g Convergence of sequences of distributions
     1.h Tensor products of distributions
     1.i Images of a distribution
  2. Fourier Transformations and Wave Front Sets
  3. Applications to Cauchy problems for Linear Partial Differential Equations
     3.1 Heat Equation
     3.2 Schrödinger Equation
     3.3 Wave/Klein-Gordon Equations
  4. Extensions of distributions.

- **Title of course:** Optimal Transport
- **Lecturers and Examiners:** Andrea Marchese, Andrea Pinamonti
- **Course:** Borrowed from the Master's degree of Trento
- **Scientific Sector:** MAT/05
- **Period:** September-December 2023
- **Venue:** University of Trento
- **Indicative number of academic hours:** 42
- **Assessment method:** seminar/oral exam
- **Contents:**
  - Weak topologies on Radon measures; compactness.
  - The Monge problem. Examples and applications. One-dimensional results.
  - The Kantorovich problem. Connections between the two formulations
  - Existence of solutions to the Kantorovich problem.
  - Theory of c-duality and c-cyclic monotonicity.
  - Kantorovic-Rubistein duality.
  - Necessary and sufficient optimality conditions.
  - Brenier theorem, Knott-Smith theorem.
  - Topology on Wasserstein space
  - Benamou-Brenier theorem
  - Gradient flow theory and applications to PDEs.
  - Remark: As the course started in September, the interested students who did not attend the course from the beginning will find on the moodle page lecture notes to study independently the first part of the course.

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- **Title of course:** Advances in Mathematical Analysis: ODE with rough vector fields
- **Lecturers and Examiners:** Andrea Marchese, Paolo Bonicatto
- **Course:** PhD course
- **Scientific Sector:** MAT/05
- **Period:** November 2023-January 2024
- **Venue:** University of Trento
- **Indicative number of academic hours:** 24
- **Assessment method**: seminar
- **Contents:** Many physical phenomena lead to consider transport equations where the coefficients are discontinuous, and therefore the classical Cauchy-Lipschitz theory breaks down. The problem to develop a theory of solutions for ODE's and transport equations was solved by Di Perna-Lions in the case of $W^{1,p}$ coefficients and by Ambrosio for BV coefficients.
  The aim of the reading seminar is to present a self contained presentation of these breakthroughs. The reading course will consist of twelve 2-hours seminars, mainly delivered by the students. The content of the reading seminar will include:
    - Classical Transport equation and continuity equation: Cauchy-Lipschitz theory
    - Uniqueness results
    - Sobolev vector fields -BV Vector fields
    - $W^{1,p}$ regularity estimates
    - Applications.

- **Title of course:** Geometric Measure Theory
- **Lecturers and Examiners:** Andrea Marchese, Paolo Bonicatto
- **Course:** Borrowed from the Master's degree of Trento
- **Scientific Sector**: MAT/05
- **Period:** March-May 2024
- **Venue:** University of Trento
- **Indicative number of academic hours:** 42
- **Assessment method:** seminar/oral exam
- **Contents**: Measure theory: measurable functions; absolutely continuous and singular measures; Radon-Nikodym and Lebesgue decomposition theorems; Riesz representation theorem; signed measures and vector measures; weak convergence of measures; operations on measures. -Differentiation of Radon Measures: Vitali and Besicovitch covering theorems; Lebesgue-Besicovitch theorem for differentiation of Radon measures on $\R^n$; Lebesgue points and density.
    - Introduction to Hausdorff measures and area and coarea formulas: Hausdorff measures and dimension; Hutchinson construction of fractal sets; Lipschitz functions; Rademacher theorem; area and coarea formulas.
    - Rectifiable sets and blow-up of Radon measures: rectifiable sets and their decomposition into Lipschitz images; approximate tangent planes to rectifiable sets; blow-up of rectifiable measures.
    - Introduction to minimal surfaces and the theory of currents. Prerequisites of multi-linear algebra, currents, mass, boundary, rectifiable currents, push-forward and product of currents, homotopy formula and flat norm, polyhedral deformation theorem, slicing, closure theorem.

- **Title of course:** Geometric Analysis
- **Lecturer:** Lorenzo Mazzieri
- **Period:** February-June 2024
- **Venue:** University of Trento
- **Course:** Borrowed from the Master's degree of Trento

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- **Indicative number of academic:** 63 (Within the 63 hours of the master's degree course, specific hours and topics will be set aside for doctoral students, to be agreed with the teacher)
- **Scientific Sector:** MAT/05
- **Examiner:** Lorenzo Mazzieri
- **Assessment method:** Oral presentation
- **Contents**:
  I) A quick introduction to differentiable manifolds, tensor bundles and tensor calculus.
  II) Covariant differentiation, Affine connections and Riemannian metrics.
  III) The Riemann curvature tensor. Fundamental properties and orthogonal decomposition. Locally conformally flat manifolds.
  IV) Fundamental equations of Riemannian Geometry. Second fundamental form. Gauss and Codazzi equations.
  V) Variational theory of the geodesics. The length functional and the energy functional. First and second variation formulas with applications.
  VI) Introduction to minimal surfaces. First and second variation of the area functional.
  VII) Comparison Geometry, Bishop-Gromov volume comparison Theorem.
  VIII)The Cheeger-Gromoll Splitting Theorem, with applications.

- **Title of course:** Advances in Mathematical Analysis and Applications. Euscotha: Euregio School on Control Theory and Applications
- **Lecturers**: Mauro Garavello, Frederic Jean, Marco Frego, Angelika Peer, Karl von Ellenrieder, Lucia Pallottino.
- **Examiners**: Fabio Bagagiolo, Andrea Pinamonti
- **Course:** PhD course
- **Scientific Sector:** MAT/05
- **Period:** February 12 – 16, 2024
- **Venue**: University of Trento
- **Indicative number of academic hours:** 24
- **Assessment method:** Oral examination/Seminar on topics of the school
- **Contents:** One week doctoral school consisting in four courses of six academic hours each:
  1) Control Problems for Conservation Laws with Applications to Traffic Flow (Garavello)
  2) Motion Planning (Jean)
  3) Robust Control with Applications to Planning and Robotics (Frego, Peer, von Ellenrieder)
  4) Optimal Control and Optimization: From single to multi-robot systems (Pallottino)
  For more information, in particular for the abstracts of the courses and the program of the school, please visit the web page https://sites.google.com/unitn.it/euscotha?usp=sharing.

- **Title of course:** Advances in Mathematical Applications to Biology and Medicine: Stability analysis of dynamical systems in mathematical biology
- **Lecturers and Examiners**: Stefania Ottaviano, Mattia Sensi
- **Course:** PhD course
- **Scientific Sector:** MAT/05- MAT/07
- **Period:** One intensive week in the second half of June 2024
- **Venue**: University of Trento
- **Indicative number of academic hours:** 20
- **Assessment method:** Seminar on selected papers
- **Contents:** Many biological and physical systems can be described by a system of nonlinear ordinary differential equations for which, differently from systems of linear differential equations, it is in generally

not possible to find explicit solutions. Thus, usually, one aims to investigate the qualitative behavior of such systems by using geometry-based methods. The course will be structured as follows. A brief introduction will be devoted to recalling basic properties of dynamical systems, in particular ODE systems, and classical methods (Lyapunov functions, LaSalle's Invariance Principle, etc.) for the stability analysis of notable solutions (equilibria and limit cycles) and trajectories. Afterwards, we will focus on more complex and recent modifications and generalizations of useful tools to study the stability of dynamical systems. Then, we will apply the theory to the study of the transient and asymptotic behavior of dynamical systems in mathematical biology. We will focus mainly, but not only, on mathematical modelling of infectious diseases, presenting various compartmental models to describe the evolution of epidemics in a population. We will introduce the concept of bifurcation analysis and apply it to the models we presented, showcasing the impact varying one parameter can have on the system under study. As a final part of the course, we will present the basis of Geometric Singular Perturbation Theory, a useful analytical technique to study systems evolving on multiple time scales.

- **Title of course:** Markov Decision Processes and Reinforcement Learning
- **Lecturers and Examiners**: Luigi Amedeo Bianchi and Francesco Cordoni
- **Course:** Borrowed from the Master's degree of Mathematics and of Physics of Trento
- **Scientific Sector:** MAT/06
- **Period:** first semester 2023/2024. For the topics covered before November 2023 (start of the doctoral course), recorded video lessons are available
- **Venue**: University of Trento
- **Indicative number of academic hours:** 48
- **Assessment method:** oral exam, oral presentation of a complementary topic or project with presentation
- **Contents:** Markov Decision Processes and their properties. Policy Evaluation and Policy Improvement with Dynamic Programming (Policy Iteration, Value Iteration). Model free learning: Monte Carlo, Temporal Differences and TD (lambda) for evaluation and control. Function approximation. Model Based Reinforcement Learning. Policy gradients. Actor Critic. Deep Reinforcement Learning.

- **Title of course**: Mathematical Physics of Quantum Relativistic Theories
- **Lecturer and Examiner:** Valter Moretti
- **Course:** Borrowed from the Master's degree of Trento
- **Scientific Sector:** MAT/07
- **Period:** Second semester 2023/2024
- **Venue:** University of Trento
- **Indicative number of academic hours:** 63 for the master's degree, but it will be reduced for PhD students.
- **Assessment method:** Oral exam
- **Contents:** The 2023-2024 program will generally cover topics in differential geometry and/or functional analysis applied to the mathematical formulation of relativistic and quantum relativistic theories: from quantum mechanics, to general relativity and quantum field theory in curved spacetime. The precise program will be set once the class of students is known.

- **Title of course:** Mathematical Physics – Differential Geometric Methods
- **Lecturers and Examiners**: Enrico Pagani, Valter Moretti, Nicolò Drago
- **Course**: Borrowed from the Master's degree of Trento
- **Scientific Sector:** MAT/07
- **Period:** February – May 2024
- **Venue**: University of Trento

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- **Indicative number of academic hours:** 66 (Within the 66 hours of the master's degree course, specific hours and topics will be set aside for doctoral students, to be agreed with the teacher)
- **Assessment method:** Lectures and oral examination at the end of the course
- **Contents:**
    - The course treats some advanced topics of Mathematical Physics, and some applications of Differential Geometry to Analytical Mechanics, Calculus of Variations, Continuum Mechanics, Special and General Relativity Theory, as indicated in the following:
    - Differential geometry, tensor calculus, fiber-bundles, connections, jet-spaces, Lie groups.
    - Analytical mechanics. Symplectic manifolds. Lagrangian and Hamiltonian formulation of Classical Mechanics. Non-holonomic constraints.
    - Symmetries of ODE and PDE and conserved quantities.
    - Geometric theory of first order partial differential equations. Wave propagation.
    - Calculus of Variation in presence of non-holonomic constraints. Geometric Optimal Control Theory. Second variation, conjugate points, Maslov theory.
    - Continuum Mechanics. Ideal and viscous fluids. Elasticity theory.
    - Basic notions of Special and General Relativity Theory.
        - B. A. Dubrovin, S. P. Novikov, A. T. Fomenko, Geometria Contemporanea, Editori Riuniti/Mir, 1987
        - W.M. Boothby, An Introduction to Differentiable Manifolds and Riemannian Geometry, Academic Press, 1975
        - V. I. Arnold, Metodi Matematici della Meccanica Classica, Editori Riuniti, 1979
        - R. Abraham, J. Marsden, Foundations of Mechanics, Benjiamin Cummings, Reading, 1978
        - R. Abraham, J. Marsden, T. Ratiu, Manifolds, tensor analysis, and applications, Springer, 2003
        - R. Courant, D. Hilbert, Methods of Mathematical Physics, Interscience Publ., 1937
        - D. McDuff, D. Salamon, Introduction to Symplectic Topology, Clarendon Press, 1988
        - S. W. Hawking, G. F. R. Hellis, The large scale structure of space-time, Cambridge U.P., 1973
        - P. Olver, Equivalence, Invariants and Symmetries, Cambridge University Press, 1995, Notes of the teacher.

- **Title of course:** Advances in Numerical Analysis and Mathematical Modelling: Multi-agent interaction models, from control to learning across scales
- **Lecturer and Examiner**: Giacomo Albi, Marco Bonafini
- **Course**: PhD course
- **Scientific Sector:** MAT/08
- **Period:** March - April 2024
- **Venue**: University of Verona (Streaming is possible)
- **Indicative number of academic hours:** 30
- **Assessment method:** Oral examination
- **Contents:** Multi-agent interaction models naturally arise in the description of large-scale systems of interacting agents appearing in economics, biology, physics, and engineering. Such systems describe the motion of a collection of individual entities at the microscopic scale interacting through simple rules.
  In the first part of the course, we will study the optimal control problem of steering a large ensemble of N agents toward the desired state. We will show that such a high-dimensional problem can be well approximated by equivalent mean-field and kinetic models when N goes to infinity. Different numerical methods for the realization of the control will be discussed as well as possible applications (opinion dynamics, evacuation problems ...). On the other side of the spectrum, we will study the problem of learning the interaction rules governing the system. In this scenario we are provided with observations of the system, and task ourself with the identification of a suitable model (and model parameters) with enough descriptive power to handle the observations. The use of different models will be discussed, ranging from classical Newtonian-like systems to game based models. We will discuss possible loss

functions to be used, and prove convergence of the learning process as the number of agents increases or as the number of repeated observations increases. Examples and applications will be presented in learning interaction kernels in classical Newtonian systems, and in learning interaction rules for systems of rational agents (e.g., pedestrians).

- Students interested in the course are invited to contact the teachers (giacomo.albi@univr.it,mauro.bonafini@univr.it).
  Streaming of the course will be possible.

- **Title of course:** Advances in Numerical Analysis: Computational aspects of complex engineering systems
- **Lecturers:** Chiara Piazzola, Franco Zivcovich
- **Examiners:** Marco Caliari, Giacomo Albi
- **Course:** PhD course
- **Scientific Sector:** MAT/08
- **Period:** March-May 2024
- **Venue:** University of Verona (Streaming is possible)
- **Indicative number of academic hours:** 24
- **Assessment method:** Oral examination: discussion of a project
- **Contents:** This course consists of two parts, each addressing a computational aspect of complex engineering systems.
  Part 1: Finite Element Methods with Python
  The first part of the course offers a dive into Finite Element Methods (FEM) using Python and its libraries Numpy and Scipy, as well as high-performance packages like PETSc. You will revisit key concepts such as sparse data structures and iterative solvers, delve into the intricacies of working within unstructured simplicial meshes, and master the assembly of Finite Elements matrices for solving Partial Differential Equations (PDEs). You will not only grasp the inner workings of FEM but also be able to tailor solutions to meet the unique challenges of the engineering problems.
  Part 2: Uncertainty Quantification for Complex Systems
  Complex engineering systems are often described by high-dimensional parametric PDEs, with the parameters being modelled by random variables to take into account e.g. their inherent variability. This part of the course addresses several approaches to assess the influence of parameter uncertainties on system behaviour, such as the Monte Carlo method, sparse grids, and the polynomial chaos expansion. You will also be introduced to multi-fidelity techniques, like the Multi-Level Monte Carlo and Multi-Index Stochastic Collocation methods, which leverage low and high fidelity model solutions to achieve the best trade-o between accuracy and computational cost. At the end, few challenging problems combining aspects from both parts of the course will be proposed to you, to give you the opportunity to put into practice what you learned.
  Students interested in the course are invited to contact the teachers: chiara.piazzola@tum.de, franco.zivcovich@univr.it

- **Title of course**: Scientific Machine Learning for Biomedical Applications
- **Lecturers and Examiners**: Sahli Costabal, Simone Pezzuto
- **Course**: Borrowed from the Master's degree of Trento
- **Scientific Sector**: MAT/08
- **Period**: May-June 2024
- **Venue**: University of Trento
- **Indicative number of academic hours**: 35
- **Assessment method**: Final project

- **Contents**: Many machine learning applications require large volumes of data to achieve accurate predictions. On the other hand, mathematical models that can accurately predict various phenomena in areas such as electrophysiology and biomechanics. This course aims at blending machine learning techniques with mathematical models to get the best of both worlds, especially when data is scarce but there is knowledge of the physics of the problem. This is the case of biomedical applications. The second part of the course will focus on modeling aspects, in particular cardiac electrophysiology. Upon completing the course, students will be able to calibrate complex mathematical models to experimental data using machine learning techniques.

- **Title of course:** Advances in Operation Research
- **Lecturer and Examiner:** Romeo Rizzi
- **Course:** PhD course
- **Scientific Sector:** MAT/09
- **Period:** Jan-Feb 2024
- **Venue:** University of Trento (but goes out also in streaming if students from Verona subscribe)
- **Indicative number of academic hours:** between 20 and 26
- **Assessment method:** get enough scores on the exercises proposed as homeworks
- **Contents**:
  - Good characterizations, recursion, and Dynamic Programming (DP)
    1 good conjectures, their flavour, and how to prove them
    2 from recursion/induction to dynamic programming
  - Introduction to Linear Programming (LP)
    1 LP problems and ILP problems, a computational complexity perspective
    2 the simplex method
    3 duality theory
    4 complementary slackness
    5 economic interpretation
  - Introduction to graphs and Combinatorial Optimization (CO)
    1 graphs and digraphs as models
    2 a few good characterizations (connectivity, Eulerian graphs, bipartite graphs, DAGs)
    3 Bellman-Ford's algorithm. Simple Temporal Networks (STNs) and scheduling.

- **Title of course:** Graphical Models and Network Science
- **Lecturer and Examiner**: Veronica Vinciotti
- **Course**: Borrowed from the Master's degree of Trento
- **Scientific Sector:** SECS-S/01
- **Period:** First semester 2023/2024. For the topics covered before November (start of the doctoral course), recorded video lessons are available
- **Venue**: University of Trento
- **Indicative number of academic hours:** 42
- **Assessment method:** Oral exam+homeworks
- **Contents:** At the end of this course, the student will be able to use graphical models to study the relation between continuous or discrete variables as well as statistical models for network data. They will learn fundamental aspects of the theory and how to apply the theory to practical cases with the use of a statistical software. In particular, the following topics will be covered:
  1. Conditional independence graphs, Markov properties and factorizations
  2. Gaussian graphical models
  3. Directed graphs and causal inference

4. Chain graphs and vector autoregressive processes
5. Modelling of network data.

- **Title of course:** Advanced Statistical methods
- **Lecturer and Examiner**: Claudio Agostinelli
- **Course**: Borrowed from the Master's degree of Trento
- **Scientific Sector:** SECS-S/01
- **Period:** Second semester 2023/2024
- **Venue**: University of Trento
- **Indicative number of academic hours:** 42
- **Assessment method:** Exam + homeworks
- **Contents:** At the end of the course, the student will learn about classical and modern techniques to study the asymptotic behaviour of estimators and of other important inferential quantitites and suitable stochastic approximations. The student will also study convergence problems through empirical processes, including convergence laws, rates of convergence and entropy measures.

- **Title of course:** Advances in Cryptography and Codes: Cryptanalysis of Code-Based Cryptosystems and beyond
- **Lecturers and Examiners**: Paolo Santini - Marche Polytechnic University (lecturer), Marco Calderini (examiner)
- **Course:** PhD course
- **Scientific Sector:** INF/01
- **Period:** January 2024 – February 2024
- **Venue**: University of Trento
- **Indicative number of academic hours:** 30
- **Assessment method:** A seminar, to be decided with the lecturers
- **Contents:** Since the proposal of the McEliece cryptosystem, many more code-based cryptosystems (i.e., cryptographic schemes based on problems which are described in terms of codes) have appeared. Schemes of this kind are normally deemed as well understood and (for most cases) are believed to be quantum-resistant. For these reasons, code-based cryptography is considered one of the most promising areas in post-quantum cryptography. Notice that the intersection between code-based cryptography and other areas in post-quantum cryptography is non trivial: for instance, many solvers for code-based problems are based on principles and procedures that are more or less ubiquitous in the cryptanalysis of post-quantum schemes.
  This course aims at presenting the most important constructions for code-based cryptosystems, with a special focus on cryptanalysis. For instance, the three alternate KEMs in the NIST 4th round will be considered, aiming to highlight the main differences between the corresponding approaches (algebraic vs MDPC vs random codes, null vs non null decryption failure probability) as well as their weaknesses.
  The course will also consider recent constructions and problems, aiming to identify interesting open questions and research directions. Notice that the topics will not confined to the context of coding theory. For instance, Zero Knowledge proof systems will be considered, as well as problems other than code-based ones (e.g., lattice-based).